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Technical report: Signals for Joint Communications and Positioning under Time-Varying Channels

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Abstract: A key aspect to design an Orthogonal-Frequency-Division-Multiplexing (OFDM) system for combined positioning and high-data-rate communications is to find optimal data and pilot power allocations. Previously, a capacity maximizing design for combined design of data and pilot for positioning and communication for static channels has been investigated. Considering the time variations of the channel and correlation between the corresponding channel taps from different OFDM symbols increase the capacity and at the same time improve the time-delay estimation accuracy. In this paper, we propose a method for joint design of data and pilot considering the effect of channel variations from one OFDM symbol to another OFDM symbol. Numerical results show that considering the effect of time variations in the joint design of data and pilots increase the capacity and improves the time-delay estimation simultaneously.

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Executive Summary

This document describes the design of OFDM sequences for positioning and communications in time-varying and frequency selective channels. First, we find an expression for joint CRB of time-delay and channel coefficients under time-varying assumption. Second, the expression for the ergodic capacity under partially known channel state information in the transmitter has been proposed. Finally, joint design of OFDM sequences for a given time-delay estimation accuracy based on maximization of the ergodic capacity is proposed. Also, the effect of time-varying channels on pilot design in time and frequency and increasing the capacity has been investigated.

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List of Acronyms and Abbreviations

Term	Description
ECRB	expected Cramér-Rao bound
CSI	channel state information
OFDM	Orthogonal frequency division multiplexing
CRB	Cramér-Rao bound

1. Introduction

The design of combined positioning and communication systems that can perform well in terms of high-data-rate transmission and delay estimation accuracy is a challenging problem. In general, the signals used for one purpose perform poorly in the other case.

To date different approaches are proposed for the signal design for channel estimation that leads to equi-power and equi-space pilots [1-3]. Pilot design for carrier frequency offset (CFO) estimation and joint estimation of channel and CFO is proposed in [4] and [5] respectively. However, joint data and pilot design based on channel and time-delay estimation is received a little attention.

Recently, joint data and pilot designs based on CRB and ECRB of time-delay and channel coefficients is investigated by solving the relaxed optimization and masking the relaxed solution to allow subcarriers to be used either for estimation or data transmission [6,7]. However, the methods are not taking into account the effect of channel variations with time that usually happens in reality, and only consider the designs for static channels. In other words, they assume that channel coefficients are the same for different OFDM symbols. Specially, in the LTE systems one needs to design an optimal positioning reference signal that can be applied for high time-delay estimation accuracy and at the same time fulfils the requirements for high data rate communications.

In our method, we consider time-varying frequency selective channels for the joint design of data and pilots based on ECRB of time-delay and channel coefficients. First, we propose a more general model than the model used in [6,7], then we find the ECRB of time-delay and channel coefficients for the general case and reduce the bounds to the special case with the assumption on the normalized delay from the lth path as $\tau_l = \tau_1 + l - 1$ where τ_1 is the delay from the first path [6,7]. We present an approximate formula for the capacity lower bound as a function of pilot and data and ECRB of time-delay and channel coefficients and design the data and pilots using the relaxed solution that is proved to be convex at sufficiently high SNR. Finally, we limit each subcarrier to be considered as either data or pilot. The results show that the close-to-optimal solution for data transmission and time-delay and channel coefficient estimation requires the stair-wise pattern of the pilots.

2. Signal Model

We use the following signal model using the assumption that channel variations in delay subspace is much slower than channel variations in the amplitude subspace. Therefore, signal model in kth subcarrier and nth OFDM symbol after removing the guard interval and taking the DFT of the output signal would be of the following form

$$\mathbf{y} = \mathbf{B}(\mathbf{I}_K \otimes \mathbf{\Gamma}(\tau_1)\mathbf{F}_d)\boldsymbol{\alpha} + \mathbf{w},$$

in which $\boldsymbol{y}$ denotes the output signal, $\boldsymbol{B}$ is the $KN \times KN$ diagonal matrix of the input signal, $\boldsymbol{I}_K$ is the $K \times K$ identity matrix, $\boldsymbol{\Gamma}(\tau_1)$ is the $N \times N$ matrix exponentially depending on the delay τ_1, $\boldsymbol{F}_d$ is the first d columns of the discrete Fourier transform matrix, $\boldsymbol{\alpha}$ is the channel coefficient vector, and $\boldsymbol{w}$ is the zero-mean white Gaussian noise vector.

It is important to note that in this model we assume that the delay from the first path can be resolved from the delay from the other multipath components. However, we have developed the case that this is not the case and solved the problem for a more general case as well.

Note that apart from the mathematical description of the system model, here we explain the meaning behind each part and intuitively motivate our approach for the joint design of data and pilots in time and frequency. The goal is to design the diagonal matrix of input signal $\boldsymbol{B}$ for K OFDM symbols each of which with N subcarriers such that the data rate is maximized and the fundamental bound on the delay estimation is upper bounded by a desired value. However, designing the input signal to fulfil these goals leads to a non-convex optimization problem of combinatorial form which needs to be solved by relaxing the optimization problem to the convex form and finding the close to optimal solution for data and pilot design problem.

3. CRB and ECRB

In this section, we briefly explain the CRB and ECRB for the delay and channel coefficients for time-varying and frequency selective channels without going through the details of the proofs for the bounds and just by presenting the bounds and explaining them for each case.

The CRB for the delay and channel coefficients for time-varying and frequency selective channels can be written as

$$\mathrm{CRB}(\boldsymbol{\tau}) = \frac{\sigma_w^2}{2}(\underbrace{\tilde{\mathbf{Q}} - \Re[\mathbf{Z}(\mathbf{Y}^H\mathbf{P}\mathbf{Y})^{-1}\mathbf{Z}^H]}_{\boldsymbol{\Phi}})^{-1},$$

$$\mathrm{CRB}(\boldsymbol{\alpha}) = \frac{\sigma_w^2}{2}(2(\mathbf{Y}^H\mathbf{P}\mathbf{Y})^{-1} + \mathbf{Y}\boldsymbol{\Phi}^{-1}\mathbf{Y}^H).$$

Without talking about the details of each equation, we only explain intuitively each part of the above expressions. The noise power is represented by σ_w^2, the first term in the expression for the CRB of delay represents the amount of information for delay estimation while the second term represents the amount of information reduced as a result of unknown channels. Accordingly, the CRB of channel coefficients has been obtained considering the fact that it is related to the CRB of delay by the term. Finally the terms $\boldsymbol{Y}$ and $\boldsymbol{Z}$ are defined as

$$\mathrm{Y}{=}(\mathrm{I}_K \otimes [\boldsymbol{\Gamma}(\tau_1)\tilde{\mathbf{g}}_f, \ldots, \boldsymbol{\Gamma}(\tau_d)\tilde{\mathbf{g}}_f]),$$

$$\mathbf{Z}^H = \left[\mathbf{X}_1^H\boldsymbol{\alpha} \ \ \ldots \ \ \mathbf{X}_d^H\boldsymbol{\alpha}\right],$$

in which $\tilde{\boldsymbol{g}}_f$ represents an $N \times 1$ vector as a result of multiplication of the first L columns of the DFT matrix by channel coefficients, and finally

$$\mathrm{X}_l = \mathbf{Y}_l^H\mathbf{P}\mathbf{Y}.$$

To take into account the random channel coefficients and not making the design only for a specific channel coefficients, we need to introduce the new bounds on delay and channel coefficients known as expected CRB (ECRB) of delay and channel coefficients defined as

$$\mathrm{ECRB}(\boldsymbol{\tau}) \approx \frac{\sigma_w^2}{2}(\underbrace{\tilde{\mathbf{Q}}_D - \mathbf{\Phi}_R}_{\tilde{\mathbf{\Phi}}})^{-1},$$

$$\mathrm{ECRB}(\boldsymbol{\alpha}) \approx \frac{\sigma_w^2}{2}(2(\mathbf{Y}^H\mathbf{P}\mathbf{Y})^{-1} + (\mathbf{Y}^H\mathbf{P}\mathbf{Y})^{-1}\mathbf{X}^H(\tilde{\mathbf{\Phi}}^{-1} \otimes \mathbf{C}_a)\mathbf{X}(\mathbf{Y}^H\mathbf{P}\mathbf{Y})^{-1}),$$

in which the expressions are pretty similar to the CRB of delay and channel coefficients presented earlier but with applying the expectation with respect to channel coefficients and using Jensen's inequality the aforementioned bounds provide sufficiently accurate metrics for the joint data and pilot design problem as will be shown in the next sections. Note that in the expression for the ECRB of channel coefficients the term $\boldsymbol{C}_a$ represents the channel covariance matrix for a particular OFDM symbol and between different OFDM symbols so it takes into account the inter correlation between channel taps of a particular OFDM symbol and cross-correlation between different OFDM symbols.

4. Channel Capacity with Partially Known CSI

In this section, we form the lower bound of channel capacity with partially known CSI at the transmitter to be used later on as the cost function for the optimization problem for joint data and pilot design.

It can be shown that the lower bound on the ergodic capacity is of the following form

$$\overline{C}_{lb} = \frac{1}{NK}\mathbb{E}[\log_2 \det(\mathbf{I} + \mathbf{P}\mathbf{R}_e^{-1}\hat{\mathbf{H}}\hat{\mathbf{H}}^H)],$$

In which

$$\hat{\mathbf{H}}\hat{\mathbf{H}}^H = \mathrm{diag}\{(\mathbf{I}_K \otimes \mathbf{F}_d)\mathbf{a}\mathbf{a}^H(\mathbf{I}_K \otimes \mathbf{F}_d^H)\},$$

and

$$\mathbf{R}_e = \mathbf{P}\mathbf{C}_{\tilde{\mathbf{H}}} + \sigma_w^2\mathbf{I},$$

where $\boldsymbol{R}_e$ depends on the error in the channel estimation and noise power and $\boldsymbol{P}$ represents the diagonal power matrix to be designed. Using the Laplace approximation by moving the expectation inside the $\log_2 det$ we obtain the approximate form of the lower bound on the ergodic capacity with partially known CSI as

$$\overline{C}_{lb} \approx \frac{1}{NK}\log_2 \det(\mathbf{I} + \mathbf{P}\tilde{\mathbf{R}}_e^{-1}\mathbb{E}[\hat{\mathbf{H}}\hat{\mathbf{H}}^H]),$$

in which $\tilde{\boldsymbol{R}}_e$ has the same expression as $\boldsymbol{R}_e$ except applying the expectation on the first term. From the proposed lower bound on the ergodic capacity with partially known CSI we interpret that the effect of error on the estimation of channel coefficients acts as a an estimation noise component and depending on the design of diagonal power matrix it can be reduced to maximize the lower bound on the ergodic capacity as will be seen in the next section.

Also, it is worth mentioning that the power matrix $\boldsymbol{P}$ needs to be designed such that the lower bound on the approximated ergodic capacity with partially known CSI is maximized for a certain upper bound on the time-delay estimation. This way the power matrix fulfil both high data rate and desired delay accuracy. However, as will be discussed in the next section the optimization problem is not of convex form and to solve the problem we need to apply the relaxation on the constraints.

5. Combined Data and Pilot Design

In this section, we formulate the optimization problem used for the pilot design for joint communication and time-delay estimation. To maximize the cost function which is the lower bound of approximate form of ergodic capacity, one needs to solve the following optimization problem

$$(P1)\begin{cases} \max\limits_{\mathbf{p}} & \overline{C}_{lb} \\ \text{s.t.} & \mathrm{ECRB}(\tau_1) \leq \epsilon \\ & \mathbf{1}^T\mathbf{p} \leq P_t \\ & \mathbf{p}_p^T\mathbf{p}_d = 0 \\ & \mathbf{p}_p \succeq \mathbf{0}; \mathbf{p}_d \succeq \mathbf{0}, \end{cases}$$

where $\boldsymbol{p}_p$ and $\boldsymbol{p}_d$ are the pilot and data power vectors. The first constraint limits the time-delay estimation accuracy by introducing a small value ε as the upper bound, the second constraint limits the total power for the design to P_t while the power vector $\boldsymbol{p}$ stands for the sum of data and pilot vectors $\boldsymbol{p} = \boldsymbol{p}_p + \boldsymbol{p}_d$, the third constraint makes the problem to be combinatorial and non-convex by using each subcarrier either as data or pilot, and finally the last constraint assures the data and pilot vectors to be non-negative. We propose the relaxed form of the problem by removing the third constraint as

$$(P2)\begin{cases} \max\limits_{\mathbf{p}} & \overline{C}_{lb} \\ \text{s.t.} & \mathrm{ECRB}(\tau_1) \leq \epsilon \\ & \mathbf{1}^T\mathbf{p} \leq P_t \\ & \mathbf{p}_p \succeq \mathbf{0}; \mathbf{p}_d \succeq \mathbf{0}. \end{cases}$$

Problem (P2) is a convex optimization problem at sufficiently high SNR since the cost function is of the form of a concave function with respect to the power vector $\boldsymbol{p}$, and the constraints are affine and convex functions of $\boldsymbol{p}$. Therefore, it can be solved using the convex optimization methods like interior point approach. After solving the relaxed optimization problem (P2) we choose the subcarriers that are used as both data and pilot as pilot if the pilot power is much stronger than data and as data if the data power is much stronger than the pilot. This way, we obtain a close-to-optimal solution for the joint time-delay estimation and communication design problem.

6. Simulation Results

In the following, we present numerical evaluations of the two dimensional combined data and pilot design for time-varying frequency selective channel. The simulation settings are in Table I where we assume 2.5ms frame including 0.5ms slots each including 6 OFDM symbols resulting 30 OFDM symbols in one frame. Further, we consider 32 subcarriers with $\Delta f = 15\ kHz$ resulting the bandwidth of $480kHz$. The SNR is defined as P_t/σ_w^2. We use the channel covariance matrix with the expression proposed in section 3.2 with allowing correlation between different channel taps for the same OFDM symbol C_a^{ii} and assuming diagonal pattern for correlation between corresponding channel taps from symbol to symbol C_a^{ij} a using the zero-order bessel function $J_0(2\pi f_D(i-j)T)$.

Parameters	Values
Number of subcarriers, N	32
Number of symbols, K	30
Number of channel taps, d	5
Normalized delay of the first path, τ_1	0.1
Normalized Doppler frequency, f_D	0.3

6.1 Joint Design

Fig 1 shows the joint design of data and pilots. The number of subcarriers and OFDM symbols are $N = 32$ and $K = 30$ respectively. Channel capacity for the combined design after masking the solution to the relaxed optimization problem (P1) is 1.1077 for the upper bound of time-delay estimation accuracy $\varepsilon = 1e - 5$ and the SNR of $27dB$. The position of pilots follow the stair-wise pattern that is what we more or less see as the Positioning Reference Signal (PRS) in the LTE systems.

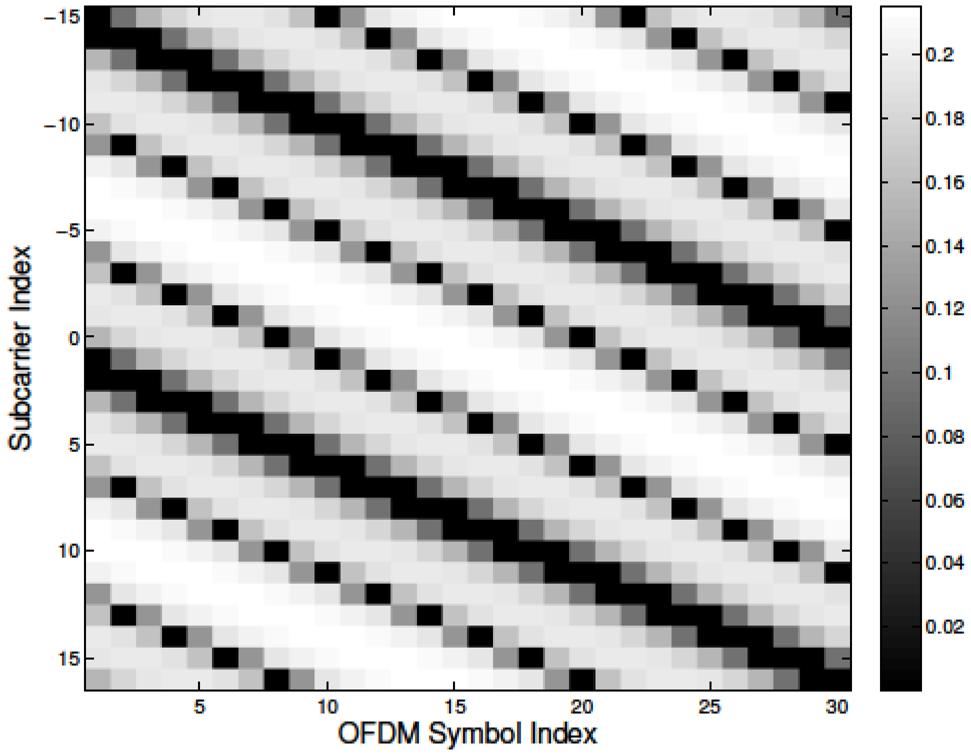

Fig. 1. Joint Data and pilot allocations in a block-fading time-varying OFDM system.

6.2 Comparison Study

In this part, we compare the behavior of optimization problems (P1) and (P2), then the behavior of the optimization with and without considering the time-delay estimation constraint is investigated. Fig. 2 compares the lower bound of channel capacity for the different values of the upper bound of time-delay estimation accuracy parameter ε of the problems (P1) and (P2). The relaxed solution with allowing each subcarrier to be used as data and pilot provides 31% higher capacity in the saturation region of the curves comparing to the solution after masking and limiting each subcarriers to be considered as either data or pilot. The SNR used for the simulation is $SNR = 27dB$. Fig. 2 shows that by increasing the accuracy in the estimation of time-delay the capacity is reduced since more subcarriers are needed to be allocated as pilots.

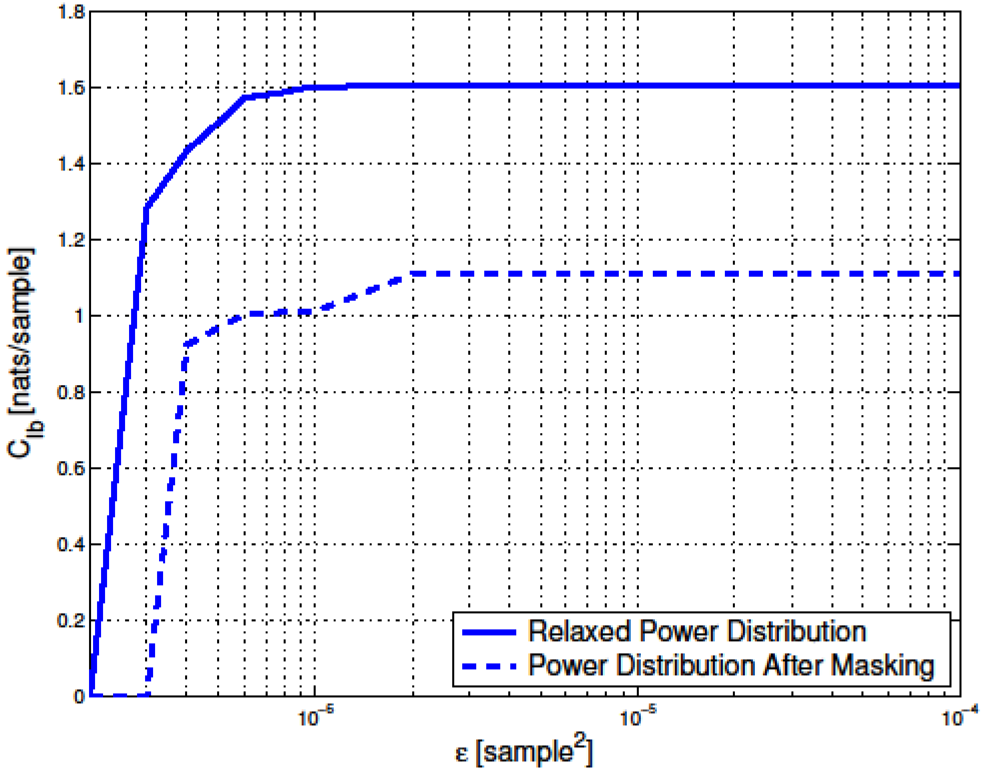

Fig. 2. Channel capacity versus the upper bound of time-delay estimation accuracy.

Fig. 3 and Fig. 4 compare the designs with and without considering time-delay constraint in terms of $ECRB(\tau_1)$ and C_{lb} for different SNR and with the upper bound on the time-delay estimation accuracy of $1e-5$ respectively. Interestingly, based on Fig. 3 after $SNR = 29dB$ the delay constraint is fulfilled without considering it in the optimization problem as a constraint. This means that by increasing the SNR, time-delay constraint in the optimization problem can be fulfilled even without directly applying it in the optimization problem. Fig. 4 shows that after $SNR = 27dB$ the values for the channel capacity with and without considering the delay constraint converge since the delay constraint is fulfilled by increasing the SNR even for the case that is not applied directly as a constraint in the optimization problem.

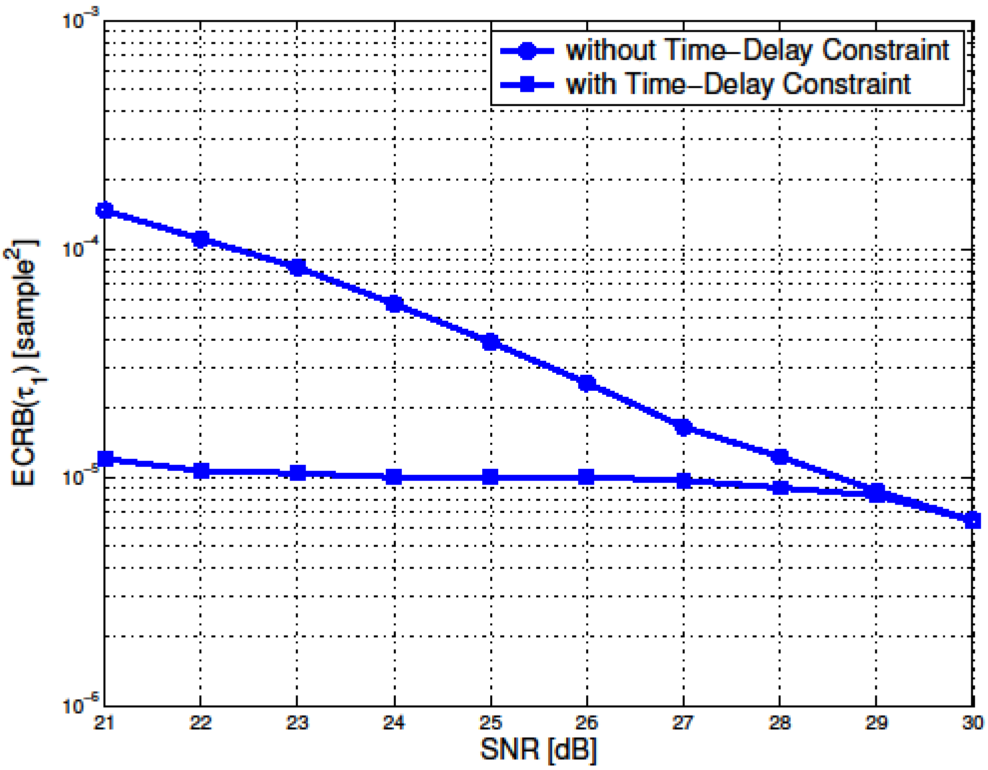

Fig. 3. ECRB of time-delay versus SNR with and without applying time-delay constraint in the optimization.

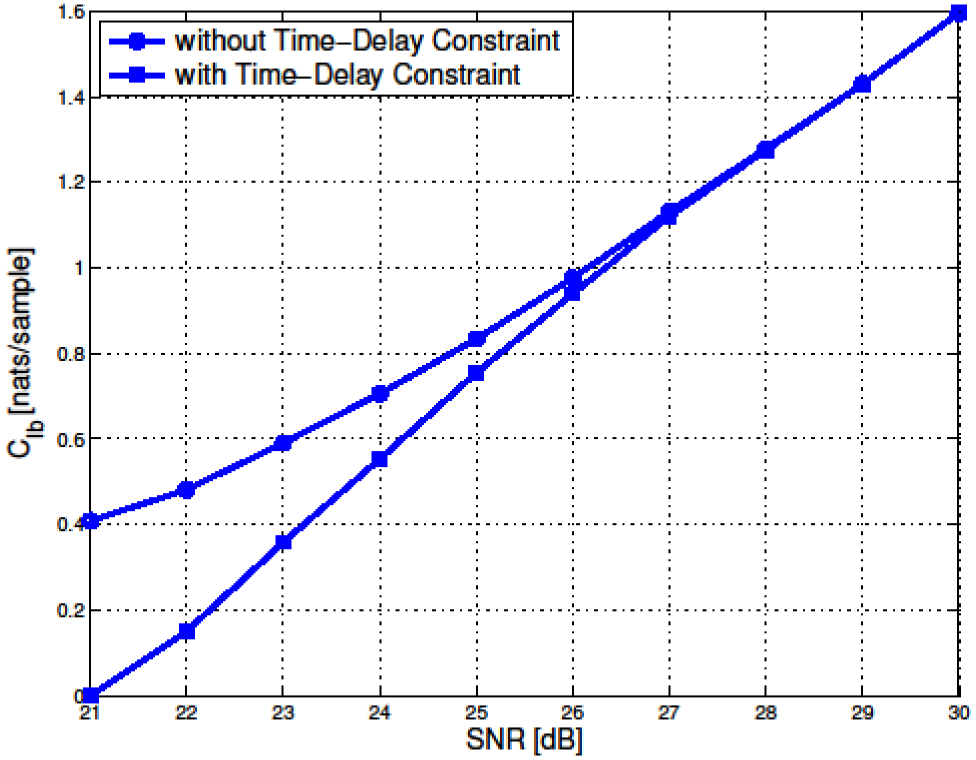

Fig. 4. Channel capacity versus the SNR with and without time-delay constraint.

7. Conclusion

Combined design of data and pilots considering the effect of channel variations for different OFDM symbols can increase the channel capacity comparing to the designs with the static channel and also improves the time-delay estimation accuracy. In this paper, the performance of near-optimal pilot and data power allocations for the case of time-varying frequency selective channels has been investigated. Results showed that joint design considering the correlation between channels taps for different OFDM symbols, higher values for the capacity and better time-delay estimation accuracy could be obtained, simultaneously.

8. References

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[7] R. Montalban, J. A. Lopez-Salcedo, G. Seco-Granados, and A. L. Swindlehurst, “Power allocation method based on the channel statistics for combined positioning and communications OFDM systems,” in *IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP)*. 2013, pp. 4384–4388, IEEE.

9. Forthcoming Research

We proposed a method for achieving high data rate and good localization accuracy for the case of time-varying frequency selective channels. This is the case for moving users that need to perform localization with sufficiently high accuracy and high data rate. The previous study was for static channels in which the movement was not taken into account. We plan to complete the research by performing more realistic simulations and submit the work as a journal paper within the next academic year preferably until June 2016.

Moreover, we developed a robust power allocation for OFDM wireless network localization to gain a certain level of localization accuracy during the agent movement. We plan to extend the work as a journal paper by comparing stochastic and robust power allocations for OFDM wireless network localization and submit the research as a journal paper within the next academic year preferably until the first of October 2015.

Finally, a novel localization method for 5G networks using the concept of millimeter wave massive MIMO has been developed that enables localization with one base station (BS). We plan to extend the work as a journal paper by considering the case for non-line of sight (NLOS) scenarios and submit the research as a journal paper within the next academic year preferably until April 2016.

10. List of Publications

[1] A. Shahmansoori, R. Montalban, J. A. Lopez-Salcedo, G. Seco-Granados, " Design of OFDM Sequences for Joint Communications and Positioning Based on the Asymptotic Expected CRB," in International Conference on Localization and GNSS (ICL-GNSS). 2014.

[2] A. Shahmansoori, R. Montalban, G. Seco-Granados, "Optimal OFDM Pilot Sequences for Time-Delay and Channel Estimation Based on the Expected CRB for a Large Number of Subcarriers," in The 15th IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC). 2014 .

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[5] Arash Shahmansoori, Gabriel E. Garcia, Giuseppe Destino, Gonzalo Seco-Granados, and Henk Wymeersch, "5G Position and Orientation Estimation through Millimeter Wave MIMO," IEEE GLOBECOM, San Diego, CA, USA, 2015.